INTERPRETING WORKED EXAMPLES OF INTEGER SUBTRACTION

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Drawing on research around the utility of worked examples, we examine how 29 first- and 27 third-grade students made sense of integer subtraction worked examples and used those examples to solve similar problems. Students first chose which of three worked examples correctly represented an integer subtraction problem and used the example to solve a similar problem. Later, we presented only the correct worked example and had them solve another similar problem. Our results highlight how their initial ideas around which worked example was correct supported or constrained their later interpretation and use of the correct worked example. Students were attuned to the number of jumps shown in the examples; however, they sometimes misinterpreted the jumps' direction. Students' visual answers were correct more than their written answers, suggesting further attention to visuals could support students' reasoning.

Keywords: Number Concepts and Operations, Cognition, Elementary School Education

Students with strong schemas for a particular concept may be more resistant to changing these schemas, even with instruction; for example, upper elementary students who more frequently used a limited addition schema (i.e., A + B = C) had difficulty solving equivalence problems (i.e., A + B = ___ + D) even after being shown correct solutions to the problems (McNeil & Alibali, 2002). To help students revise their existing schemas, many studies described the use of worked examples as a support that can effectively promote middle-school students' increased conceptual understanding (Booth et al., 2013), especially for students with lower prior knowledge (Atkinson et al., 2000; Schwartz et al., 2016). Worked examples can engage students in productive struggle, drawing students' attention to relevant features in their current schemas or important underlying features when extending their schemas and highlight alternative ways of thinking about problems (Booth et al., 2015; Lange et al., 2014). Yet, students with lower prior knowledge might not know which features are relevant to pay attention to (Booth & Davenport, 2013; Crooks & Alibali, 2013). With integer operations, there are many features for students to pay attention to, and their use of them can vary greatly depending on their number schemas (e.g., Aqazade & Bofferding, 2019; Bofferding, 2019). In this study, we further explore how elementary students, who attend to different problem features based on their schemas, interpret integer worked examples and investigate how their prior knowledge schemas correspond to their application of worked examples.

Theoretical Framework

From a blended theory of conceptual change perspective (Scheiner, 2020), students' understanding of negative integers and operations changes through an interaction between their number schemas (e.g., a mental integer number line, Bofferding, 2014; see also Case, 1996, McNeil & Alibali, 2002) and pieces or features that comprise the schemas (e.g., order, value, symbols, operations, Bofferding, 2019; see also Booth & Davenport, 2013; Case, 1996; Crooks & Alibali, 2013). Students might solve 3 – 5 in many different ways, depending on their integer schemas and interpretation of the problem features. Some students might interpret the problem as 5 – 3 (Bishop et al., 2014; Bofferding, 2010) due to a positive integer schema that you cannot

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subtract a larger number from a smaller number (Karp et al., 2014) together with a flexible interpretation of the order of the features in the problem (i.e., a student could read the problem as three taken away from five). Students who understand that order matters in subtraction but have a strong whole number schema might pay attention to the features, start with three, and argue that the answer is zero or that you cannot take away more than three (Bishop et al., 2011, 2014; Bofferding & Wessman-Enzinger, 2017). Finally, students who have an integer schema might count back from three and answer negative two (Aqazade et al., 2016; Bishop et al., 2011).

Including negative numbers within the problems themselves may cause additional struggles for students. For example, when solving integer subtraction problems, such as -2 – 4, students need to distinguish the minus sign feature appended to the two (i.e., negative sign or unary meaning of the minus sign, Vlassis, 2004, 2008) from the minus sign feature between the two and four (i.e., subtraction sign or binary meaning of the minus sign, Vlassis, 2004, 2008). In fact, students who attend to subtraction signs and reason based on a whole number schema might ignore the negative sign or interpret it as an indication to subtract two (Bofferding, 2019). Other students might think the negative sign needs to be part of the answer and append it to the answer after solving 4 – 2, getting -2 (e.g., Bofferding, 2010; Bofferding, 2019). Students who know the order of negative integers, might still struggle with interpreting their value; therefore, they may vary in whether they subtract by getting numbers smaller in absolute value, counting toward zero and answering "2," or by getting numbers smaller in linear value and answering "-6" (Ball, 2013; Bofferding, 2019).

When analyzing worked examples, students with particular schemas might look for particular features that align with their schemas in order to make judgments about why worked examples are correct or not or to apply ideas from a worked example to a similar problem that they need to solve themselves. In this study, we add to previous literature by focusing in particular on the ways that elementary students interpret and use integer subtraction worked examples, highlighting what features (e.g., number of jumps, direction) students use and their reasoning when making use of the worked examples. Our research questions include:
How do first and third graders make use of integer subtraction worked examples?

1. How do they interpret and determine a correct worked example?
2. Which features are important to them when they try and solve a similar worked example?

Methods

Participants, Setting, and Data Sources

Twenty-nine first-grade and 27 third-grade students from a public elementary school in the midwestern United States with 9% English-Language-Learners and 46% economically disadvantaged participated in this study. As a part of a larger study, students completed two worked example tasks involving integer subtraction about one month apart. The first worked example task included three potential solution strategies for 3 – 5 associated with a number path model: one correct (B) and two incorrect: 3 – 5 = 0 (C) and 3 – 5 = 2 (A) (see Figure 1).

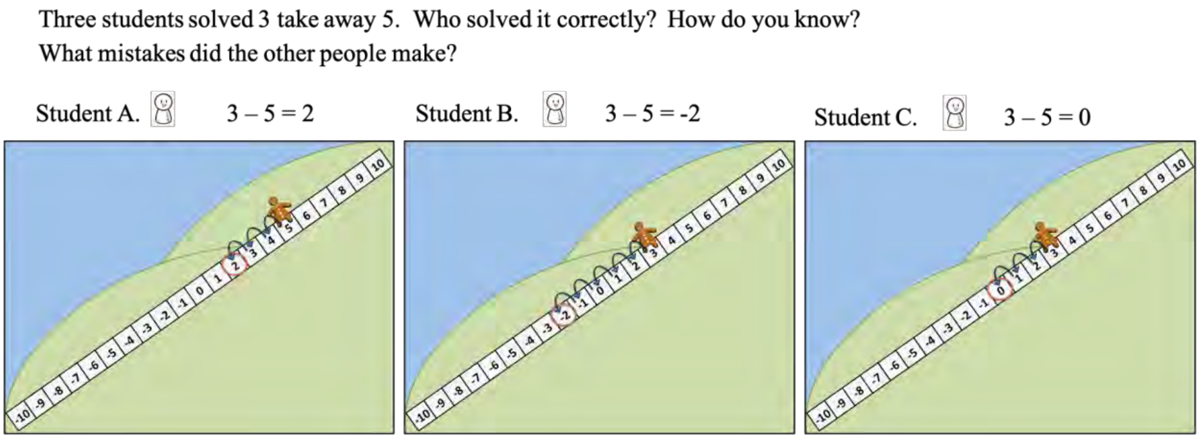

Figure 1: First Worked Example Task

Students were asked to choose the worked example that is correct, explain their reasoning, and describe the mistakes in the other two examples. Next, without receiving feedback on their response, we encouraged them to use that example to solve 1 – 4 and use an empty number path model to draw their solution (see Figure 2A). The second worked example task (see Figure 2B) showed students the correct solution for 3 – 5 = -2 illustrated on a number path. Next, students were asked to solve -2 – 4 and draw on an empty number path using this worked example.

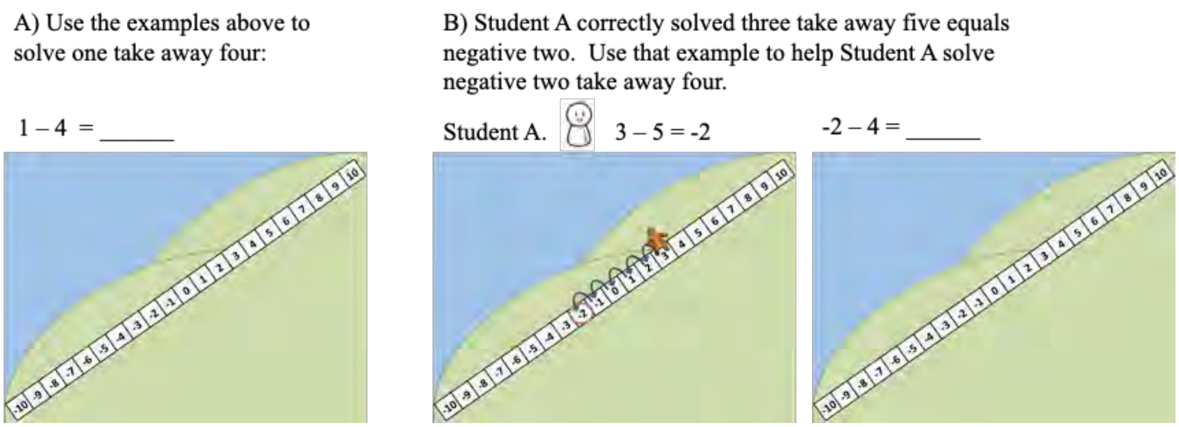

Figure 2: A: First Worked Example Task (cont.) and B: Second Worked Example Task

Data Analysis

On the first worked example task, we determined the number of students who chose A, B, or C as the correct worked example for 3 – 5. Next, we determined if they solved 1 – 4 by using a similar strategy to A from the worked example (i.e., reversed the order of the numbers and answered 3), B from the worked example (i.e., counted through zero and correctly answered -3), C from the worked example (i.e., stopped at zero and answered 0) or *other* (i.e., an answer not aligned with one of the three strategies presented in the first worked example or if their picture did not match their written answer). Then, we noted whether their strategies for both problems matched (e.g., did they answer zero for 1 – 4 if they had selected C for 3 – 5). Finally, we analyzed the explanation of their choice and identified elements or features they focused on, including number of jumps, starting number, ending number, or direction.

On the second worked example task, we calculated the number of students answering correctly and if their written answers matched with the number they landed on on the number path or the number they circled. To better understand their strategies, we analyzed their drawings based on the direction of jumps, starting number, number of jumps, ending number, and circled number. Lastly, we looked for any patterns between the first and second worked example tasks in students' solution strategies.

Findings

First Worked Example Task

The majority of students at both grade levels chose 3 – 5 = 2 (A) or 3 – 5 = -2 (B) as the correct worked example for solving 3 – 5 (see Table 1). When applying this example, overall, 17% of first graders and 44% of third graders correctly solved 1 – 4 as represented in their written answer; however, 38% of first and 48% of third graders showed the answer correctly through their drawing on the empty number path. Among these students, only 14% of first and 37% of third graders solved 1 – 4 correctly both on the written response and on the number path.

Table 1: Students' Choice of Correct Worked Example for 3 – 5 and Solutions to 1 – 4

		First Worked Example Task			
	First Graders (*n*=28) [a]	**1 – 4 → 3**	**1 – 4 = -3**	**1 – 4 → 0**	**1 – 4 → other**
(A)	**3 – 5 = 2** (*n*=9, 31%)	2 (22%)	4 (44%)	0 (0%)	3 (33%)
(B)	**3 – 5 = -2** (*n*=14, 48%)	0 (0%)	6 (43%)	0 (0%)	8 (57%)
I	**3 – 5 = 0** (*n*=5, 17%)	1 (20%)	1 (20%)	1 (20%)	2 (40%)
	Third Graders (*n*=27)	**1 – 4 → 3**	**1 – 4 = -3**	**1 – 4 → 0**	**1 – 4 → other**
(A)	**3 – 5 = 2** (*n*=7, 26%)	2 (29%)	1 (14%)	1 (14%)	3 (43%)
(B)	**3 – 5 = -2** (*n*=16, 59%)	1 (6%)	11 (69%)	0 (0%)	4 (25%)
I	**3 – 5 = 0** (*n*=4, 15%)	1 (25%)	1 (25%)	2 (50%)	0 (0%)
	Overall (*n*=55) [a]	**1 – 4 → 3**	**1 – 4 = -3**	**1 – 4 → 0**	**1 – 4 → other**
(A)	**3 – 5 = 2** (*n*=16, 29%)	4 (25%)	5 (31%)	1 (6%)	6 (38%)
(B)	**3 – 5 = -2** (*n*=30, 54%)	1 (3%)	17 (57%)	0 (0%)	12 (40%)
I	**3 – 5 = 0** (*n*=9, 16%)	2 (22%)	2 (22%)	3 (33%)	2 (22%)

Note. The table shows the number and percent of students who picked a particular worked example (i.e., A, B, or C) who also solved 1 – 4 in a particular way. For example, in the first row with the nine first graders who thought the worked example A (3 – 5 = 2) was correct, two of those students (or 22% of those selecting A), also solved 1 – 4 with an answer of 3. [a] One first grader did not choose a worked example for 3 – 5, so this student was not included in the data presented here.

Choosing A: 3 – 5 = 2 and Applying This Worked Example to Solve 1 – 4

Students' explanations for why they chose A as the correct worked example were often focused on the number of jumps or reinterpreting the subtrahend as being three (seeing the problem as 5 – 3), which was coupled with referencing the starting and ending number (or

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answer) in the problem. For example, Horse2$_{(3rd)}$ focused on the expected answer and the subtrahend as being three and said, "Took away three and they had two." Bat6$_{(1st)}$ chose to focus on the subtrahend as three and the starting number as five, "He's on five, and he only has to move three spaces." Likewise, when students then explained why B and C were not the correct worked examples for solving 3 – 5, they also pointed out the starting number, ending number, or the number of jumps. For example, Rabbit5$_{(3rd)}$ said, "He put a minus two" for B and she "did five minus three–one, two, three, so it's two." Robin3$_{(1st)}$ chose A because "he jumped three" and B was not correct because "he was supposed to jump three."

Out of 16 students who chose 3 – 5 = 2 (A), only four students' (two first and two third graders) responses to 1 – 4 were aligned with this choice. They made use of 3 – 5 = 2 the same way and applied it to their strategy for 1 – 4 to answer 3. Interestingly, five students (four first-grade and one third-grade) solved 1 – 4 correctly, which corresponds to 3 – 5 = -2 (B). One third grader's answer to 1 – 4 represented the incorrect 3 – 5 = 0 worked example as they answered 0. Even though Sheep6$_{(3rd)}$ wrote "3" and said, "One take away four," when drawing on the number path, her movements showed that she interpreted 1 – 4 as -1 + 4, perhaps because she misinterpreted the direction of the arrows on the examples: "Because they're subtracting one minus four, and I thought you started at the minus one and you go up to the three like those two [options B and C]." Finally, five students' solution strategies for 1 – 4 were not aligned with their choice of 3 – 5 = 2 and indicated either making an exact copy of 3 – 5 = 2, doing 1 + 4, putting the number of jumps as the answer (i.e., 4), or jumping the wrong number on the number path. We classified these types of responses as *other*.

Choosing B: 3 – 5 = -2 and Applying This Worked Example to Solve 1 – 4

Similar to students who incorrectly chose A as the correct worked example for 3 – 5, students with the correct choice of B also referred to the number of jumps, starting number, and ending number when explaining why A and C were incorrect worked examples and B was correct. For instance, Robin4$_{(1st)}$ counted the number of jumps to justify her choice, "One, two, three, four, five" and for C, said, "It didn't get far enough." Duck3$_{(1st)}$ explained why A was not the correct worked example, "They started on the five and landed on two." He, for B, counted from -2 to 3 and confirmed it was 5 and said, "I think it would be in the minuses." Finally, for C, he referred to the answer, "It's not on the zero, it's not on the minus."

Out of 30 students with the choice of 3 – 5 = -2 (B), 17 students' (six first and 11 third graders) responses to 1 – 4 were aligned with this choice. They correctly made use of the 3 – 5 = -2 worked example, wrote -3 for 1– 4, and correctly showed their solution on the number path. Nine students (six first and three third graders) made use of the 3 – 5 = -2 worked example to some extent when solving 1 – 4. Some of them started at an incorrect number (e.g., 0 or 3) but counted backward the correct number of jumps to get into the negative numbers. Some other students answered 1 – 4 correctly when using the empty number path but said the answer was three and wrote three on their paper. Only one third grader—Goat2$_{(5th)}$—despite choosing B for the correct worked example of 3 – 5, answered 3 for 1 – 4 seeing it as 4 – 1. Two first graders' responses to 1 – 4 did not align with their choice of B because they started at an incorrect number and jumped an incorrect amount. Finally, one third grader skipped this problem.

Choosing C: 3 – 5 = 0 and Applying This Worked Example to Solve 1 – 4

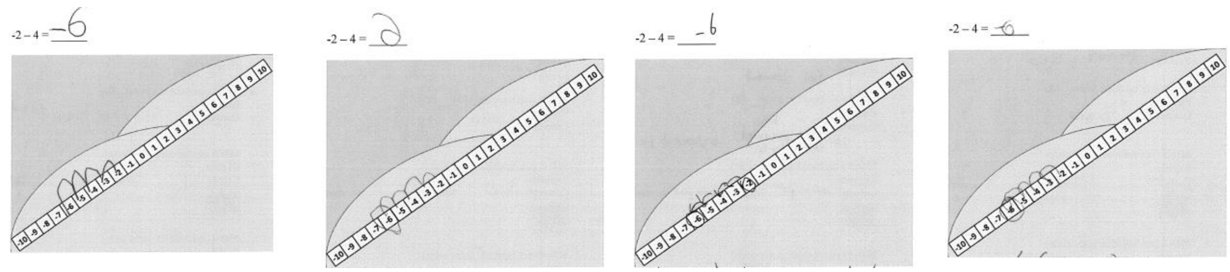
Interestingly, rather than primarily referring to the number of jumps, students who chose C for the correct worked example of 3 – 5 often focused on the starting or ending number. As an example, Goose9$_{(1st)}$ rejected A and B because their answer was not zero. Duck1$_{(1st)}$ used his

fingers to take away five from three and, similar to Goose9$_{(3rd)}$, thought A and B were incorrect worked examples because “It only gets two” or “Three minus five is not two, it’s zero.”

Out of nine students choosing 3 – 5 = 0, only two third graders and one first grader applied the same strategy when solving 1 – 4 and answered zero. One first and one third grader answered -3, corresponding to the correct 3 – 5 = -2 worked example. One first and one third grader solved 1 – 4 as 4 – 1 reflecting the incorrect 3 – 5 = 2 worked example. One first grader—Bat5—used her fingers to count and answered 0. However, when drawing on the number path, she correctly showed -3 as the answer for 1 – 4. Lastly, one first grader did not provide any answer for 1 – 4.

Second Worked Example Task

On the second worked example task, 33% of first and 54% of third graders correctly solved -2 – 4 on the written response (43% overall), but even more students solved it correctly using the empty number path (52% of first and 65% of third graders; 58% overall). In fact, 43% of students (33% first and 54% third graders) correctly answered on both the written response and the number path because if they wrote the correct answer, they also illustrated it correctly. Among these students, seven (four first and three third graders) had chosen 3 – 5 = 2 (A), 13 (four first and 11 third graders) had chosen 3 – 5 = -2 (B), and three (one first and two third graders) had chosen 3 – 5 = 0 (C) in the initial worked example task. These students’ drawings on the empty number path indicated that many correctly identified the important features of the 3 – 5 = -2 worked example including the starting number and number of jumps to use when solving -2 – 4 on an empty number path (see Figure 3 for examples). However, many did not show the ending number (or answer) by circling it or circled both the starting and ending numbers. Some of them did not show the directional movements on their jumps. Thus, from only drawings, it was not clear where the starting and ending numbers were and which direction they jumped. In fact, Rabbit3$_{(3rd)}$ only put a mark on -6 and explained, “I went to four [-4] and one, two [referring to the jumps],” and she ended at -6.



Duck2$_{(1st)}$ showed non-directional jumps and did not circle the answer.

Goose3$_{(1st)}$ showed non-directional jumps and circled the answer.

Horse4$_{(3rd)}$ showed directional jumps and circled the start and answer.

Horse8$_{(3rd)}$ showed directional jumps and circled the answer.

Figure 3: Examples of Students Solving -2 – 4 Using an Empty Number Path

Of those students who chose 3 – 5 = 2 (A) on the first worked example task and did not answer -2 – 4 correctly, a few answered two–solving it as 4 – 2—or negative two—solving the problem as 2 – 4 (see Horse9$_{(3rd)}$ in Figure 4). Their drawings on the empty number path demonstrated that they often started at an incorrect number (e.g., copying the exact 3 – 5 worked example and starting at 3) or jumped in an upward direction (see Figure 4 for more examples).

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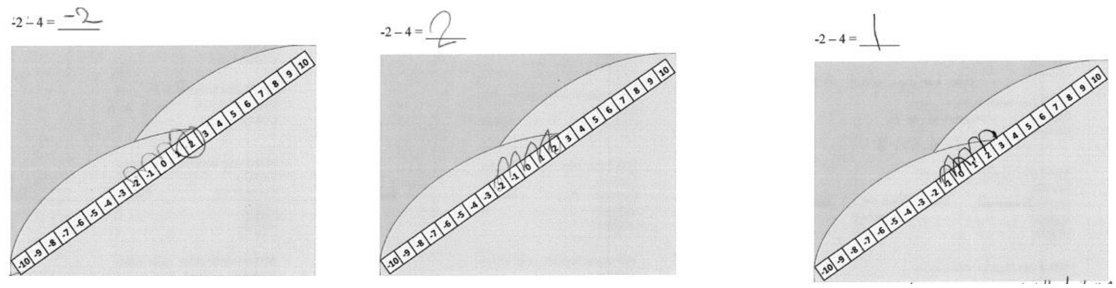

Horse9$_{(3rd)}$ started at 2 and jumped downward. | Goose4$_{(1st)}$ jumped in an upward direction. | Bat6$_{(1st)}$ started at 3 as in the example but jumped 6 (using the 2 and 4) by going down and up.

Figure 4: Examples of Students Solving -2 – 4 Incorrectly

One first grader—Robin3—correctly made use of the worked example when using the empty number path, saying, "Start up at negative two, and three, and four, and six [ending at -6]" but wrote down six as his answer. Among students who chose 3 – 5 = -2 (B) on the first worked example task, seven (four first and three third graders) also correctly made use of the second worked example task when using the empty number path but did not answer correctly on the written response. For example, Robin2$_{(1st)}$ said, "Take away four," drew four jumps between -2 and -6 on the number path, but said, "It's six" and wrote "6."

Other common answers of -2 – 4 among the students with the choice of B included -2, 2, and 0. Students' strategies for -2 – 4 as shown in their drawings on the empty number path often resulted in an incorrect answer because they started at 2 and jumped downward 4 to get to -2 or started at -2 and jumped 4 upward to get to 2. An interesting example was Goose6$_{(1st)}$ who started at -2 and made jumps to 4. She then counted the jumps (or distance) between -2 and 4; however, she did not take account of the direction and answered 6.

Some of the students who chose 3 – 5 = 0 I on the first worked example task and were incorrect on -2 – 4 responded 0 or 2. For example, Bat5$_{(1st)}$ said, "Of course I'll have to start at 2" but actually started at -2 on the number path and justified starting at -2 because, "It showed me (pointing to the -2 in the problem)." Then, she made four jumps upward, "One, two, three, four" and said, "It equals two." After she wrote 2, the interview asked her why she went up, and she referred back to the worked example, misinterpreting it by explaining, "Because this one, it says 3 – 5 equals negative two, and I saw that you had to go up instead of down." Goose9$_{(1st)}$ answered 0 on the written response because she used her fingers; she held up two fingers and then put them down when trying to take away four. To model this, on the empty number path, she correctly started at -2 but then jumped upward twice and stopped at 0.

Discussion and Implications

When given the option to choose which worked example correctly illustrated 3 – 5, students' inclination to choose 3 – 5 = 2 (A) indicates that about a third of the students had a strong prior schema for subtraction as subtracting a smaller number from a larger one (i.e., 5 – 3) (e.g., Bishop et al., 2014; Bofferding, 2011; Murray, 1985). Their whole number subtraction schema was strong enough that even when presented with the correct example (worked example B; 3 – 5 = -2), they did not determine it as a match. However, when then asked to solve 1 – 4, a few of

these students—especially first graders—were able to answer correctly by starting at the correct initial number. These results suggest that introducing negative numbers as a result of subtraction (i.e., 3 – 5 = -2) could support students in developing a directional interpretation of subtraction and weaken (or eliminate) the schema that you can only subtract a smaller number from a larger one.

Students often focused on the number of jumps but, especially on the first worked example task, aligned their interpretation of jumps to how they viewed the problem (e.g., if they interpreted the problem as 5 – 3, they talked about it as having three jumps); sometimes they misinterpreted the direction of the jumps in the worked example, which was more prevalent with the second worked example task (-2 – 4). Students might have counted from -2 to -1, 0 and so on instead of -2 to -3, -4, and so on to align with interpretations of subtraction as getting smaller in absolute value (because it wouldn't make sense for them to go in a direction where the numbers were increasing in absolute value). Thus, students might need more experience interpreting and using number path and number line visuals, which could support their developing understanding of integer order and values and help those students who primarily relied on using their fingers and thought the answer to the first worked example task was zero.

In our previous work, we found that many students would solve integer subtraction problems by ignoring the negative signs, subtracting the number with smaller absolute value from the number with larger absolute value, and then append a negative sign to their answer (e.g., Aqazade et al., 2018); in this case, students would solve -2 – 4 as 4 – 2 = 2 and then make the answer -2. However, we did not see any students use this strategy, suggesting both their focus on the jumps and use of the worked example visuals helped them avoid this misinterpretation.

Overall, encouraging students to make sense of and use the integer worked examples provides opportunities for productive struggle and potential to resolve those challenges over time. Particularly, such encouragement in using the visual as presented in the worked example did help the students because they had higher performance on the visuals than when writing numerical answers. Part of the difference between the two formats is that students who were not familiar with negative numbers did not include the negative sign in their written answers, even if they landed at a negative number. Therefore, the tasks also revealed what elements or features students interpreted as important and provided insight into their number schemas. Further, our work adds to our understanding of the usefulness of worked examples (e.g., Booth et al., 2013; Booth & Davenport, 2013); by the second subtraction worked example task, the first graders' performance was closer to the third graders, so the worked examples seemed to help the novices begin to make sense of the problems.

Acknowledgments

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